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Fermions Tunneling and Entropy Correction of Black Hole in Gravity's Rainbow Space Time

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Abstract Based on the fermions tunneling method, correction to Bekenstein-Hawking entropy of black hole in gravity's rainbow space time is discussed. We consider not only the quantum corrections in the single particle action revealing that these are proportional to the usual semiclassical contribution but also the quantum effects of space time arising from the change of energy of probe particles moving in it. The result shows that as the high order terms with respect to $\hbar$ of the action is considered, the first and second corrections, namely the logarithmic term and the inverse area term respectively, are produced. This result is consistent with that of loop quantum gravity and other entropy correction theories.

Keywords Hawking radiation · Fermions tunneling · Gravity's rainbow · Bekenstein-Hawking entropy · Correction

1 Introduction

The research on thermodynamics of black hole has been one of the most important contents in black hole physics. Since Stephen Hawking first proved that black holes had thermal radiation [1], many efforts have been devoted to studying it from the static, stationary, and non-stationary black holes in subsequent decades [2–9]. During all the methods, the most significant method is the radial null geodesic method, proposed by Parikh and Wilczek in 2000 [10] that treated Hawking radiation as a tunneling process from the classically forbidden trajectory. As self-gravitational interaction and back reaction of radiation were taken

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into account, it was found that the thermal spectrum was not pure thermal and the tunneling rate satisfied the underlying unitary theory. In addition to the null geodesic method, there is another method which was first developed by Padmanabhan et al. [11]. In this method, the Hawking radiation is derived by calculating the particles' classical action from the Hamilton-Jacobi equation. Now it is believed that the tunneling radiation provides not only an alternative conceptual means for understanding the actual emission process of black hole but also a useful verification of black hole thermodynamics. To confirm this tunneling theory, Kerner and Mann soon after put forward that one also can obtain the tunneling temperature of fermions by the Dirac equation [12]. The major problem of this method is also how to find the action of radiation. Until now, tunneling of scalar and Dirac particles have been extended to a series black hole space time [11–29]. However all of them are confined to the semiclassical approximation and do not consider quantum corrections. Correspondingly, the entropy in these cases is

$$S = \frac{c^3 \kappa_B A}{4G\hbar}, \tag{1}$$

where c, κ_B, G, $\hbar$, A are speed of light, Boltzmann's constant, Newton's gravitational constant, the reduced Plank's constant and the area of the horizon. In fact, about quantum corrections to black hole entropy, there are many models and methods [30–34]. It has been proved that the general formalism of black hole entropy is

$$S = \frac{c^3 \kappa_B A}{4G\hbar} + \alpha \ln \frac{c^3 \kappa_B A}{4G\hbar} + O\left(\frac{G\hbar}{c^3 \kappa_B A}\right) + \text{constant}, \tag{2}$$

where α is a model-dependent (dimensionless) parameter. In the case of Loop Quantum Gravity [34], α has been fixed at $\alpha = -1/2$. In String Theory [33], the sign of α depends on the number of field species appearing in the low energy approximation. Very recently, Refs. [35, 36] have successfully discussed entropy correction of scalar particles and Ref. [37] even involved the Dirac particles. In all these cases, the key skill is the expansion of action of radiation with respect to $\hbar$. After considering the high order terms to $\hbar$ of action, one can obtain the corrected Hawking temperature and the corrected entropy via the second law of black hole thermodynamics. However, all these cases are confined to the static space time, and none of them consider the quantum correction arising from the background space time.

In this paper, we attempt to discuss the entropy correction of black hole in gravity's rainbow from the viewpoint of fermions tunneling. We take the modified Anti-de Sitter Schwarzschild black hole, which stems from the incorporation of doubly special relativity (DSR) with the framework of general relativity [38], as an example. The DSR has gradually been viewed as an effective theory to describe physics phenomena at extremely high energy level when the semi-classical effects of quantum gravity is taken into account and it also has been thought as a potential candidate to solve the conflict between Plank scale physics and Lorentz symmetry. The main feature of DSR incorporating with the curvature of space-time is that the geometry of space time depends on the energy of a particle moving in it. That is to say, for the space time with Planck scale correction effects arising from DSR, there are different geometries of space time for particles with different energies. The Schwarzschild solution of the rainbow metric was first presented in [39] and recently Ref. [40] successfully extended it to the Anti-de Sitter case with the consideration of cosmological constant in the Einstein field equation. In that paper, the thermodynamics properties of modified Anti-de Sitter Schwarzschild black hole have been investigated. Here we further discuss the Dirac particle tunneling beyond semiclassical approximation by employing the Dirac equation from this background geometry.

The remainders of this paper begin with a brief introduction of the modified Anti-de Sitter Schwarzschild black holes from the rainbow gravity theory, including its thermodynamics properties. Then in Sect. 3, we employ the fermions tunneling method beyond semiclassical approximation to discuss its tunneling temperature and the corresponding entropy correction. Finally, a briefly discussion is given in Sect. 4.

2 Introduction to Anti-de Sitter Schwarzschild Black Hole in Gravity's Rainbow

In rainbow gravity with a cosmological constant, the Einstein field equation is

$$G_{\mu\nu}(E) = 8\pi G(E)\Gamma_{\mu\nu}(E) + g_{\mu\nu}(E)\Lambda(E). \tag{3}$$

From this, the modified Anti-de Sitter Schwarzschild black hole solution in gravity's rainbow is given as [40]

$$ds^2 = -Adt_s^2 + B^{-1}dr^2 + r^2/f_2^2 d\Omega^2, \tag{4}$$

where $A = (1 - \frac{2m}{r} + r^2/L^2)/f_1^2(E;\lambda)$, $B = f_2^2(E;\lambda)(1 - \frac{2m}{r} + r^2/L^2)$, m is the mass of the modified black hole, $f_1(E;\lambda)$ and $f_2(E;\lambda)$ are energy functions which satisfy

$$E^2 f_1^2(E;\lambda) - p^2 f_2^2(E;\lambda) = m_0^2, \tag{5}$$

which is a modified form of the usual energy momentum relation and in which E is total energy that measured at infinity, λ is a parameter of order the Planck length. From (5), it is obvious that the modified Anti-de Sitter Schwarzschild black hole is energy dependent. That is to say, a given infinity observer with different energy quantum to probe the space time will obtain different space time geometries. Because of this, the present space time is endowed with a Plank scale modification and has some quantum effects [41].

On the other hand, The Arnowitt-Deser-Misner (ADM) mass of this black hole can be calculated via the Komar integral

$$M = \frac{-1}{8\pi}\int \xi_{(t)}^{\mu;\nu} d^2\Sigma_{\mu\nu} = \frac{m}{f_1 f_2}, \tag{6}$$

where $\xi_{(t)}^{\mu}$ is the normalized time-like Killing vector. Obviously, for the modified Anti-de Sitter Schwarzschild black holes, the ADM mass is not equal to the mass parameter m but depends on the energies of the probe particles. This is because the quantum correction effect of the gravity's rainbow space time that stems from deformed special relativity (DSR) which has energy dependence. For the metric (4), we can obtain the event horizon r_h as the usual Schwarzschild Anti-de Sitter black hole for all observers.

According to the definition of surface gravity

$$\kappa = -\frac{1}{2}\lim_{r\to r_h}\sqrt{-\frac{g^{11}}{g^{00}}}\frac{(g^{00})'}{g^{00}}, \tag{7}$$

one can straightforward obtain the surface gravity for modified Anti-de Sitter Schwarzschild black holes as

$$\kappa = \frac{f_2(E;\lambda)}{2f_1(E;\lambda)}\left(\frac{1}{r_h} + \frac{3r_h}{L^2}\right), \tag{8}$$

which gives the modified Hawking temperature

$$T=\frac{1}{2\pi}\kappa=\frac{f_2(E;\lambda)}{4\pi f_1(E;\lambda)}\left(\frac{1}{r_h}+\frac{3r_h}{L^2}\right). \tag{9}$$

Obviously, the temperature of the modified black holes is different for probes with different energy. As t_s is constant and $r=r_h$, we also can calculate the horizon area of the black hole

$$A_h=\int dA=\int\sqrt{g}d\theta d\varphi=4\pi\frac{r_h^2}{f_2^2}, \tag{10}$$

where, the determinant is determined by

$$g=\begin{vmatrix} g_{22} & g_{23}\\ g_{32} & g_{33}\end{vmatrix}=\frac{\sin^2\theta}{f_2^4}r_h^4. \tag{11}$$

From above, one can easily find that the mass, the temperature and the entropy obey the differential and integral forms of the first law of black hole thermodynamics.

3 Entropy Corrections via Fermions Tunneling Method

In this section, we concentrate on discussing entropy correction of the Anti-de Sitter Schwarzschild black hole from the fermions tunneling point of view. We first find the corrected Hawking temperature by the Dirac equation

$$\gamma^\mu D_\mu\psi=0, \tag{12}$$

where the Greek indices $\mu,\nu=0,1,2,3$, D_μ being the spinor covariant derivative defined by

$$D_\mu=\partial_\mu+\Omega_\mu,\quad \Omega_\mu=\frac{1}{2}i\Gamma^{\alpha\beta}_\mu\Sigma_{\alpha\beta},\ \Sigma_{\alpha\beta}=\frac{1}{4}i[\gamma^\alpha,\gamma^\beta]. \tag{13}$$

To solve the Dirac equation, we should first find the γ^μ matrix. Based on the relation $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}I$, we pick for them as

$$\begin{aligned}
\gamma^t&=\frac{1}{\sqrt{A}}\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix}, &\quad \gamma^r&=\sqrt{B}\begin{pmatrix} 0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix},\\
\gamma^\theta&=\frac{f_2}{r}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}, &\quad \gamma^\varphi&=\frac{f_2}{r\sin\theta}\begin{pmatrix} 0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix}.
\end{aligned} \tag{14}$$

Another necessity to solve the Dirac equation is the employment of wave function. It is well known that the major work of tunneling paradigm is to find the imaginary part of the action for the process of s-wave emission across the horizon, which in turn is related to the Boltzmann factor for emission at the Hawking temperature according to the semi-classical WKB approximation. Therefore a felicitous relation between the wave function and action of radiant particles should be found. Here we make the following ansatz

$$\psi_\uparrow(t,r,\theta,\varphi)=\begin{bmatrix} A(t,r,\theta)\xi_\uparrow\\ B(t,r,\theta)\xi_\uparrow\end{bmatrix}\exp\left[\frac{i}{\hbar}I_\uparrow(t,r,\theta)\right], \tag{15}$$

$$\psi_\downarrow(t,r,\theta,\varphi)=\begin{bmatrix} C(t,r,\theta)\xi_\downarrow\\ D(t,r,\theta)\xi_\downarrow\end{bmatrix}\exp\left[\frac{i}{\hbar}I_\downarrow(t,r,\theta)\right], \tag{16}$$

in which $I_{\uparrow/\downarrow}$ being actions of emitted spin up or down particles and $\xi_{\uparrow/\uparrow}$ being eigenvectors of σ^3. In this paper, we are only interested in the case of spin down particles owing to the calculation of the spin up is the same as this case other than some changes of the sign.

On the other hand, since for radial trajectories only the $(r-t)$ sector of the metric (4) is important, (12) in this case hence can be expressed as

$$i\gamma^i\partial_i\psi - \frac{1}{2}(g^{tt}\gamma^i\Gamma^r_{it} - g^{rr}\gamma^i\Gamma^t_{ir})\Sigma_{rt}\psi = 0, \tag{17}$$

where $i = 0, 1$. Substituting the above ansatz (16) into this equation, we find terms relating to the action are

$$\frac{iC}{\sqrt{A}}\partial_t I_\downarrow - D\sqrt{B}\partial_r I_\downarrow = 0, \tag{18}$$

$$\frac{iD}{\sqrt{A}}\partial_t I_\downarrow + C\sqrt{B}\partial_r I_\downarrow = 0. \tag{19}$$

For the action, if we discuss it in the semi-classical region, implying the high order term to $\hbar$ are omitted, the general Hawking temperature will be produced. As to the temperature correction here, the high order term to $\hbar$ can't be omitted, which means the action, $I_\downarrow = I$, should be

$$I(r,t) = I_0(r,t) + \sum_i \hbar^i I_i(r,t), \tag{20}$$

where$i = 1, 2, 3, \ldots$. In this expansion the terms from $O(\hbar)$ onwards are treated as quantum corrections over the semiclassical value I_0. Solving (18) and (19), we find $C = \pm iD$, implying the functional form of the above individual sets of linear differential equations is the same. Therefore the action I_i is dependent on I_0, that is I_i's are proportional to I_0. Since I_0 has the dimension of $\hbar$, the coefficients thus should have the dimension of inverse of $\hbar$. In the units $G = c = \kappa_B = 1$, the Planck constant $\hbar$ is of the order of square of the Planck length l_p. Therefore, the proportionality constants have the dimension of r_h^{-2}, and (27) hence can be written as

$$I(r,t) = I_0(r,t) + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}} I_0(r,t) = \left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right) I_0(r,t), \tag{21}$$

where β_i's are dimensionless constant parameters. To obtain the action it hence enough to solve I_0, which satisfies (18) and (19). Considering the existence of time-like killing vector $(\frac{\partial}{\partial t})^a$, we perform the following variable separation

$$I_0 = -\omega t + W(r), \tag{22}$$

where ω is a real constant which represents the emitted particle's energy. In this case, (18) and (19) can be rewritten as

$$\frac{iC\omega}{\sqrt{A}} + D\sqrt{B}W(r)_{,r} = 0, \tag{23}$$

$$\frac{-iD\omega}{\sqrt{A}} + C\sqrt{B}W(r)_{,r} = 0, \tag{24}$$

here $W(r)_{,r} = \partial_r W(r)$. These two equations have two possible solutions that take the form as

$$C = iD, \qquad W(r) = \int \frac{\omega}{\sqrt{AB}} dr, \tag{25}$$

$$C = -iD, \qquad W(r)_{,r} = -\int \frac{\omega}{\sqrt{AB}} dr, \tag{26}$$

where $+/-$ sign correspond to the outgoing/ingoing mode. As (22) and (21) are taken into account, the action is

$$I(r,t) = \left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right)\left(-\omega \pm \int \frac{\omega}{\sqrt{AB}} dr\right). \tag{27}$$

The ingoing and outgoing probability of particles hence are

$$P(in) = \exp\left[\frac{2}{\hbar}\left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right)\left(\omega \mathrm{Im} t + \mathrm{Im} \int \frac{\omega}{\sqrt{AB}} dr\right)\right], \tag{28}$$

$$P(out) = \exp\left[\frac{2}{\hbar}\left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right)\left(\omega \mathrm{Im} t - \mathrm{Im} \int \frac{\omega}{\sqrt{AB}} dr\right)\right]. \tag{29}$$

In the classical limit, there is no reflection and everything is absorbed by black hole, the ingoing probability $P(in)$ hence has to be unity, leading to

$$\omega \mathrm{Im} t = -\mathrm{Im} \int \frac{\omega}{\sqrt{AB}} dr. \tag{30}$$

The probability of the outgoing particle therefore can be written as

$$P(out) = \exp\left[-\frac{4}{\hbar}\left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right)\mathrm{Im} \int \frac{\omega}{\sqrt{AB}} dr\right]. \tag{31}$$

According to the principle of detailed balance

$$P(out) = \exp\left(-\frac{\omega}{T_h}\right)P(in), \tag{32}$$

one can easily get the Hawking temperature

$$T_h = \left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right)^{-1} T_H, \tag{33}$$

where $(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}})$ are the corrections due to the quantum effect and $T_H = \frac{\hbar}{4}(\mathrm{Im} \int \frac{dr}{\sqrt{AB}})^{-1}$ is the standard semiclassical Hawking temperature which equals to that in (9).

Next, we continue discussing the entropy corrections. It is well known that the differential form of the second law of black hole thermodynamics is

$$dM = T_h dS_{BH} + \Phi dQ + \Omega dJ. \tag{34}$$

As to the Schwarzschild Anti-de Sitter black hole, the entropy can be expressed as

$$S_{bh} = \int \frac{dM}{T_h} = \frac{4\pi}{\hbar f_2^2} \int \frac{r_h L^2}{L^2 + 3r_h^2} \left(1 + \sum_i \beta_i \frac{\hbar^i}{r_h^{2i}}\right) dm. \tag{35}$$

In addition, according to the null hypersurface equation, the horizon r_h satisfies

$$r^3 + L^2 r - 2mL^2 = 0. \tag{36}$$

And its differential takes the form as

$$3r^2 dr + L^2 dr - 2L^2 dm = 0. \tag{37}$$

Inserting this equation into (35), one get

$$\begin{aligned} S_{bh} &= \int \frac{dM}{T_h} = \frac{\pi}{\hbar f_2^2} \int \left(2r_h + 2\beta_1 \frac{\hbar}{r_h} + 2\beta_2 \frac{\hbar^2}{r_h^3} + \cdots\right) dr \\ &= \frac{A}{4\hbar} + \frac{\pi}{f_2^2} \beta_1 \ln A - 4\pi\beta_2 \hbar \frac{1}{f_2^4 A} + \cdots + O(\hbar) \\ &= S_{BH} + \frac{\pi}{f_2^2} \beta_1 \ln S_{BH} - \frac{\pi\beta_2\hbar}{f_2^4} \frac{1}{S_{BH}} + \cdots + O(\hbar). \end{aligned} \tag{38}$$

We can see that the first term is the usual semiclassical area law, and the other terms are the quantum corrections, which mainly contain two parts: the logarithmic term in S_{BH} and the inverse area term of S_{BH}. We note that the logarithmic correction term has been also obtained

by other approaches [42, 43], and in some literatures [44, 45], the coefficient of the logarithmic correction term is controversial. In our result, it is determined by the dimensionless constant β_1. It also should be noted that in this case, the entropy is dependent on the energy of probe particles and the correction parameters, namely those of the logarithmic term and inverse area term, are related to the background geometry too.

4 Conclusions

We have investigated the entropy correction of Anti-de Sitter Schwarzschild black hole in gravity's rainbow via fermions tunneling method beyond semiclassical approximation. We found as the high order terms with respect to $\hbar$ of the action is considered, the first and second corrections, these are the logarithmic term and the inverse area term respectively, are produced. Owing to the space time geometry depends on the energy of probe particles, we find the correction parameters are also related to the probe quantum. In the low energy region i.e. $E/E_P \ll 1(E_P = \sqrt{1/8\pi}$ is the plank energy), the correspondence principle requires that f_1 and f_2 approach to unit, our result is consistent with that in [36]. Of course as pointed in [37], we also can find the correction parameters by trace anomaly.

We should point that a major problem in this paper is how to determine the proportionality constants of I_i to I_0. In [36], the authors considered it as m^{-2}. However, this choice can not extend to the other space time, leading one can't find the correction terms. Here we consider the Planck constant $\hbar$ is of the order of square of the Planck length l_p and the proportionality constants have the dimension of r_h^{-2}. Our choice is more convenient and can further extend to the charged and rotating cases, which is our next research.

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